A Price-Based Unit Commitment Model Considering Uncertainties with a Fuzzy Regression Model

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Abstract- **In a regulated environment, unit commitment refers to optimizing generation resources to satisfy load demand at least cost. But in a deregulated power market, generation companies have no obligation to satisfy customers' demands. Instead, they seek their maximum profits. This paper proposed a price-based unit commitment algorithm considering uncertainties for generation companies to maximize their profits. Due to uncertainties existing in energy prices, a fuzzy regression model is proposed to handle these uncertainties. Based on the fuzzy energy price model, an algorithm considering uncertainties of energy prices solved by cooperation of the greedy algorithm and quadratic program is presented in this paper. The New York electrical market data are used to verify this algorithm, and test results of the 5-unit system and 36-unit system are presented in this paper.**

Keywords- Deregulated Power Market; Price-based Unit Commitment; Fuzzy Regression Model; Greedy Algorithm; Quadratic Program

I. INTRODUCTION

For the vertically regulated power industry, unit commitment (UC) refers to optimizing generation resources to satisfy load demand at least cost. But in the deregulated power market, generation companies (GENCOs) run UC not for minimizing operation costs but for maximizing profits for their own. Satisfying load is no longer an obligation for the GENCOs, and security would be unbundled for energy and worked as an ancillary service. The GENCO can only consider a schedule that obtains the maximum profits neglecting the load demand.

Many methods [1-9] have been proposed for the traditional cost-minimization unit commitment. In the competitive environment, UC plays a new role to maximize GENCOs' profits. At the stance of GENCOs, new formulations have been proposed to solve this problem. In [10], the authors formulate a profited-based unit commitment considering power and reserve generating. A hybrid method between Lagrange Relaxation and Evolutionary Programming is applied to solve this new UC problem. In [11], the authors describe a genetic algorithm solution to the new profit-based unit commitment problem.

In [12], the authors formulate the price-based unit commitment (PBUC) problem based on the mixed integer programming (MIP). The PBUC solution by utilizing MIP is compared with that of Lagrangian relaxation method. Test results show the efficiency and advantages of the MIP methods for solving PBUC. In [13], a security-constrained price-based unit commitment (SPUC) algorithm underlining the coordination technique which is implemented in two stages is proposed. This paper describes a coordination process between GENCOs and the ISO for congestion management and reducing the risk of failure to supply loads. At first, GENCOs apply priced-based unit commitment (without transmission security constraints), schedule their generating units and submit their energy and capacity bids to the ISO for maximizing their revenue. The ISO obtains transmission information as well from TRANSCOs via OASIS. Using the two sets of generation and transmission data, the ISO executes inter-zonal and intra-zonal congestion management and contingency analysis to minimize line flow violations and the risk supplying loads. In [14], a methodology based on dynamic programming and enumeration has been proposed to solve the price based unit commitment problem. The technique has further been extended to optimize the problem for a group of generators.

Because energy prices are unknown before bidding, uncertainties of energy prices are important factors to build the new UC problem. Therefore, this paper presents a price-based UC problem considering uncertainties of energy prices at the stance of GENCOs. A fuzzy regression model is used to take uncertainties exiting in energy prices into account. The New York electrical market data are adopted to verify this model. This problem is solved by the greedy algorithm and quadratic program using the concept of decommitment [9]. For GENCOs, the unit may be turned off in some on-line periods only if doing so is cost-saving and would not cause infeasibility. Given a feasible unit commitment, this algorithm uses the greedy algorithm to determine an optimal strategy for decommitting overcommitted units. This paper is organized as follows. The energy price model is described in Section II. Section III will introduce the price-based unit commitment formulation. The greedy algorithm and quadratic program used to solve this problem are specified in Section IV. Finally, test results of the 5-unit and 36-unit systems are given in Section V.

II. THE ENERGY PRICE MODEL

Due to uncertainties of the energy prices, forecasts of energy prices are essential information to the GENCOs. In this paper, a fuzzy regression model [15] is used to forecast the range of energy prices, and a method to build the energy price scenarios of the next day's twenty-four hours is proposed according to the forecast ranges of energy prices.

The most important factor to the energy price is the load demand. One of the most popular methods for representing the relation between these two sets of data is regression analysis. In regression analysis, the relation between parameters x and y is represented by the following polynomial equation:

$$y = a_0 + a_1 x + a_2 x^2 + \ldots\ldots + a_m x^m \tag{1}$$

By choosing the order m we can represent nonlinear relations. Parameters are determined so that the distance (or error) between an observed data point and its corresponding point on the polynomial will be minimal.

Fig. 1 shows the demand-price data of the New York electrical market from January 2002 to May 2005. When the

demand is low, the relation between the price and demand is much like linearity. However, as the demand increases, the trend pushes up the price. Therefore, a linear regression model is applied to low demand data, while higher order curves are more suitable in representing high demand data.

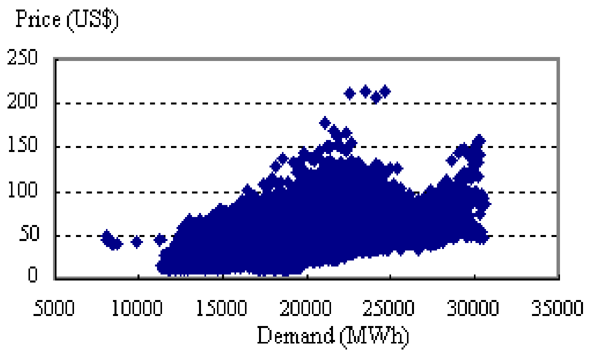

Fig. 1 The demand-price data of the New York electrical market

The TSK-fuzzy model was developed by Takagi, Sugeno, and Kang [16-17] to represent the nonlinear relation of multiple inputs and output data, based on the format of fuzzy reasoning. In this paper, we divide the data clusters of demand into two overlapping regions-low demand and high demand-such as shown by the membership functions of fuzzy sets low and high in Fig. 2.

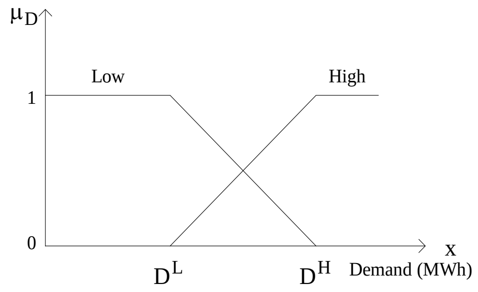

Fig. 2 Fuzzy sets to represent low and high demand

The low and high data clusters are supposed to overlap between D^L and D^H. Linear and quadratic regression curves are applied to low and high demand data respectively. Such a fuzzy model may be represented by the following rules:

$$\begin{aligned}&\text{IF } x \text{ is low THEN } y_1 = a_0^{(1)} + a_1^{(1)}x\\&\text{IF } x \text{ is high THEN } y_2 = a_0^{(2)} + a_1^{(2)}x + a_2^{(2)}x^2\end{aligned} \tag{2}$$

where in this case x represents the demand and y_i, i=1, 2 stand for electricity prices.

The composite nonlinear input-output relation is obtained by

$$y = \frac{\sum \mu_i \cdot y_i}{\sum \mu_i} \tag{3}$$

where μ_i is the degree of membership of the demand data belonging to either the low (i=1) or high (i=2) fuzzy set.

When x is smaller than D^L, linear regression is solely applied. When x is greater than D^H, only quadratic regression is applied. When x falls in between, both equations are employed with the continuously varying degree of weight μ_i.

A. Fuzzy Regression Model

In the fuzzy regression model, the parameters in (1) are replaced with fuzzy numbers as shown in (4) to cover a wide range of data.

$$Y = A_0 + A_1 x + A_2 x^2 + \cdots + A_m x^m \tag{4}$$

The parameters A_0, A_1,….are so determined that the observed data are encompassed by the fuzzy regression model, and the resultant left-hand variable Y is also a fuzzy number, which has a region of data covered in a varying degree of possibility. Fig. 3 shows the triangular fuzzy set representing the fuzzy number A_i with three crisp parameters, namely a_i, c_i^+, c_i^-.

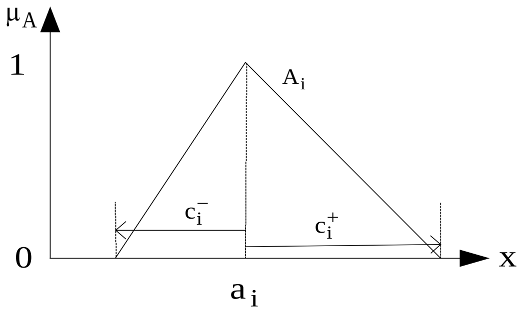

Fig. 3 Triangular fuzzy set to represent regression parameters

Here, a_i is the most likely value of the regression parameter, whereas c_i^+ and c_i^- are possible maximum spread from a_i to the higher and lower values of the parameter, respectively. We use the expression $A_i = \langle a_i, c_i^+, c_i^- \rangle$ to represent such a triangular fuzzy number hereafter. In the modeling process, the mean value a_i of the fuzzy number is most simply determined by conventional regression, but, naturally, it can also be obtained by other methods such as neural networks.

To determine the remaining parameters for the fuzzy numbers (c_i^+ and c_i^-), a linear programming model is used to fit the model to the given data within as small an area as possible. The optimization process is formulated as follows:

$$\begin{aligned}
&\text{Minimize}\\
&\sum_{t=1}^{n}\left\{c_0^+ + c_1^+ x(t) + \cdots + c_m^+ x(t)^m + c_0^- + c_1^- x(t) + \cdots + c_m^- x(t)^m\right\}\\
&\text{S.T.}\\
&a_0 + a_1 x(1) + a_2 x(1)^2 + \cdots + a_m x(1)^m + c_0^+ + c_1^+ x(1) + c_2^+ x(1)^2 + \cdots + c_m^+ x(1)^m \geq y(1)\\
&\quad\vdots\\
&a_0 + a_1 x(n) + a_2 x(n)^2 + \cdots + a_m x(n)^m + c_0^+ + c_1^+ x(n) + c_2^+ x(n)^2 + \cdots + c_m^+ x(n)^m \geq y(n)\\
&a_0 + a_1 x(1) + a_2 x(1)^2 + \cdots + a_m x(1)^m - c_0^- - c_1^- x(1) - c_2^- x(1)^2 - \cdots - c_m^- x(1)^m \leq y(1)\\
&\quad\vdots\\
&a_0 + a_1 x(n) + a_2 x(n)^2 + \cdots + a_m x(n)^m - c_0^- - c_1^- x(n) - c_2^- x(n)^2 - \cdots - c_m^- x(n)^m \leq y(n)\\
&c_0^+, c_0^-, c_1^+, c_1^-, \cdots, c_m^+, c_m^- \geq 0
\end{aligned} \tag{5}$$

where n is the total number of observed data points.

In order to narrow the range of the fuzzy regression model, some extreme market data that have little chance happening are eliminated. The modified price-demand data are shown in Fig. 4. Applying these data to the above model, parameters for the low and high demand are described in Table I and II.

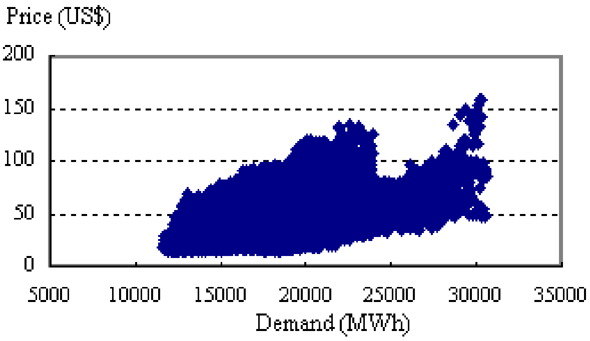

Fig. 4 The modified demand-price data of the New York electrical market

TABLE Ⅰ PARAMETERS FOR LOW DEMAND MODEL

	1st Order	Constant
a	3.27484×10^{-3}	-13.23068
c^+	3.3579×10^{-3}	0
c^-	1.7781×10^{-3}	0

TABLE Ⅱ PARAMETERS FOR HIGH DEMAND MODEL

	2nd Order	1st Order	Constant
a	1.57089×10^{-7}	-1.708595×10^{-3}	-1.70383×10^{-3}
c^+	7.32589×10^{-8}	0	0
c^-	4.97407×10^{-8}	0	0

Parameters for Table I and II are built by the demand-price data from January 2002 to May 2005. Using these parameters and the demand data of June 1 to 7, energy prices of June, 1 to 7 can be estimated as shown in Fig. 5.

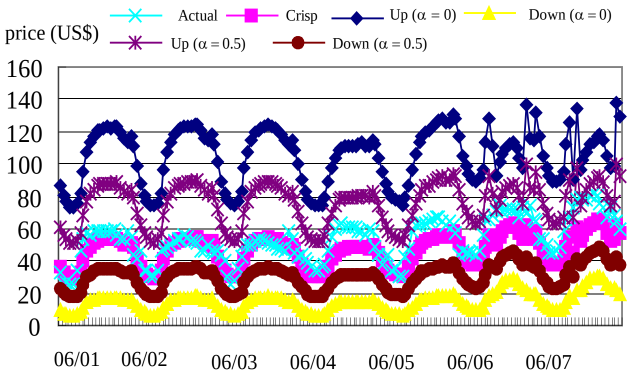

Fig. 5 Energy prices estimated by fuzzy regression model

Even after reasonable data modification, the resultant range of fuzzy model may appear too broad because the optimization process introduced above considers the entire possible range of the observed data. One idea to narrow down the focus in an integral fashion is to apply $\alpha-$cuts of the fuzzy numbers in (4) to modify their shape. Using a single parameter α ($0\le\alpha\le1$), the base of the triangular fuzzy numbers are modified as the equation (6). At $\alpha=0$, we have the original fuzzy regression model. As the value of α increases toward 1, the range encompassed in the fuzzy model narrows, and eventually it shrinks to the convention regression model at $\alpha=1$. For $\alpha>0$, some of the data points outside the range of the fuzzy model are neglected, and the number of missed data will increase as α approaches to 1. Therefore, the value of α can be interpreted as indicating the willingness of the decision-maker to accept the risk of missing data points to obtain a narrow range.

$$A_i=\left\langle a_i, c_i^+\times(1-\alpha), c_i^-\times(1-\alpha)\right\rangle \tag{6}$$

B. *Energy Price Scenarios*

Due to uncertainties of energy prices, different energy price scenarios are considered in the price-based unit commitment model. The energy price scenarios are established according to the estimated energy price range. They are built by choosing α value from 0.5 to 1.0 with the step of 0.1. Except $\alpha=1.0$ with one curve, each α value corresponds to two curves. Therefore, 11 energy price scenarios are created. Each energy price scenario is given a probability proportional to its α value by the following equation.

$$P_s=\frac{\alpha_s}{\sum_{i=1}^{S}\alpha_i} \tag{7}$$

where

P_s is the probability of the energy price scenario s,

α_s is the α value of the energy price scenario s,

S is the number of energy price scenarios.

Fig. 6 shows the 11 energy price scenarios of June 1, 2005 according to the fuzzy regression model. The probability for each curve is calculated by equation (7). These energy price scenarios will be used by the proposed price-based unit commitment model.

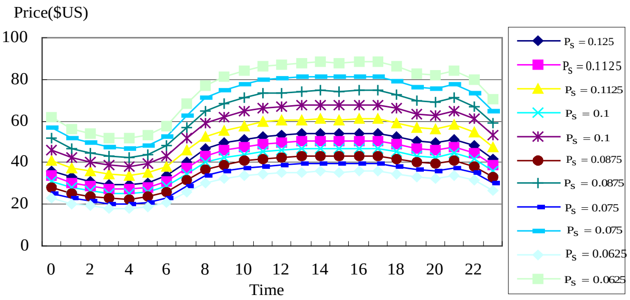

Fig. 6 Possible energy prices scenarios (2005/06/01)

III. THE PRICE-BASED UNIT COMMITMENT MODEL

This new price-based UC problem is different from the traditional cost-minimization UC. The goal of the price-based unit commitment is to maximize the profit (i.e., revenue minus cost) of GENCOs subject to all constraints. In the deregulated environment, energy prices are unknown when GENCOs propose their bids. Therefore, this new algorithm should consider uncertainties existing in energy prices and the objective function is to maximize profit instead of cost minimization. Besides, the new UC problem doesn't consider satisfying load demand as its necessary constraint. The objective function and constraints of this model are described below. The objective function of this model is first depicted as follows.

$$\max\ \sum_{s=1}^{S}P_s*\left(\sum_{i=1}^{n}\sum_{t=1}^{T}RV(s,i,t)-TC(i,t)\right) \tag{8}$$

where

RV is the revenue of GENCO

TC is the production cost

S is the number of energy price scenarios, and P_s is the probability for each scenario.

$$RV(s,i,t) = PE(s,t) * P(i,t) * I(i,t) \quad (9)$$

$$TC(i,t) = \{C_i(P(i,t)) + St(i,t)\} * I(i,t) \quad (10)$$

where

PE(s,t) is the energy price at hour t for scenario s,

P(i,t) is energy generation of generator i at time t,

St(i,t) is the start up cost,

I(i,t) is the unit status of unit i at time t,

$$C_i(P(i,t)) = a(i) + b(i) * P(i,t) + c(i) * P(i,t)^2$$

is the production cost of unit i at time t,

S is the number of energy price scenarios, and P_s is the probability for each scenario.

From equation (9), it can be seen that the revenue is from the sales of energy. Profit is defined as the revenue minus production costs. Constraints for the new UC problem are also different from those of the traditional one. These constraints are described below.

(a) System energy limits

$$\sum_{i=1}^{n} P(i,t) * I(i,t) \le D_t, \quad t = 1, \cdots T \quad (11)$$

where

D_t is forecasted demand at hour t.

(b) Unit generation constraints

$$P_{g\min}(i) \le P(i,t) * I(i,t) \le P_{g\max}(i) \quad (12)$$

where

$P_{g\min}(i)$ is minimum generation limit of generator i,

$P_{g\max}(i)$ is maximum generation limit of generator i.

(c) Unit minimum on/off durations

$$\sum_{t}^{t+T^{on}(i)-1} I(i,t) \ge \alpha_{it} * T^{on}(i) \quad t = 1, \ldots, T - T^{on}(i) + 1 \quad (13)$$

$$\sum_{t}^{T} I(i,t) \ge \alpha_{it} * (T - t + 1) \quad t = T - T^{on}(i) + 2, \ldots, T \quad (14)$$

$$\sum_{t}^{t+T^{off}(i)-1} [1 - I(i,t)] \ge \beta_{it} * T^{off}(i) \quad t = 1, \ldots, T - T^{off}(i) + 1 \quad (15)$$

$$\sum_{t}^{T} [1 - I(i,t)] \ge \beta_{it} * (T - t + 1) \quad t = T - T^{off}(i) + 2, \ldots, T \quad (16)$$

where

α_{it} is a binary variable that is equal to 1 if unit i is started up at hour t and is 0 otherwise,

β_{it} is a binary variable that is equal to 1 if unit i is shut down at hour t and is 0 otherwise,

T is the planning time periods,

$T^{on}(i)$ is the minimum on time of generator i,

$T^{off}(i)$ is the minimum down time of generator i.

(d) Unit ramping constraints

$$P(i,t) - P(i,t-1) \le UR(i) \text{ as unit i ramps up} \quad (17)$$

$$P(i,t-1) - P(i,t) \le DR(i) \text{ as unit i ramps down} \quad (18)$$

where

UR(i) is ramp up rate limit of generator i,

DR(i) is ramp down rate limit of generator i.

This model described above for the price-based unit commitment is difficult to solve. But if 'I (i, t)' is given, this problem became a quadratic programming problem that is easy to solve. Based on this observation, this paper used a greedy algorithm creating the units' status variables 'I (i, t)', and a quadratic program is used to maximize the profit based on current 'I (i, t)'. The next section will introduce the greedy algorithm, quadratic program, as well as how to apply them to solve the price-based unit commitment problem.

IV. THE GREEDY ALGORITHM AND QUADRATIC PROGRAM

The idea of the greedy algorithm is simple. Given a set Q^t, the next element chosen is one that gives the greatest immediate increase in value, provided that such an element exists. Moreover, once an element is chosen, it is kept throughout the algorithm. Let v (Q) be a real-valued function defined on all subsets of N= {1… n} and consider the problem max $\{v(Q): Q \subseteq N\}$.The followings demonstrate the iterative process of this algorithm [18].

Initialization: $Q^0 = \phi$, t=1.

Iteration t:

Step 1: Let $j_t = \arg\max_{j \in N \setminus Q^{t-1}} v(Q^{t-1} \cup \{j\})$ with ties broken arbitrarily.

Step 2: If $v(Q^{t-1} \cup \{j_t\}) \le v(Q^{t-1})$, stop. Q^{t-1} is a greedy solution.

Step 3: If $v(Q^{t-1} \cup \{j_t\}) > v(Q^{t-1})$, set $Q^t = Q^{t-1} \cup \{j_t\}$.

Step 4: If $Q^t = N$, stop. N is a greedy solution. Otherwise let t=t+1.

Where $v(Q)$ is a real-valued function defined on all subsets of N={1,……,n}.

In order to specify how to apply the greedy algorithm cooperating with quadratic program to solve the price-based unit commitment problem, Table III shows an example of the 5-unit system. Fig. 7 shows the flow diagram of the new algorithm for the price-based unit commitment problem. The description of this algorithm is given below.

TABLE III PARAMETERS FOR THE 5-UNIT SYSTEM

Unit	P_{max}	P_{min}	a(i)	b(i)	c(i)	Min on	Min off	Ini T	Ramp Up	Ramp Dn	ST MBtu
1	12	2.4	24.3891	25.5472	0.02533	1	-1	-1	12	12	0
2	100	25	217.8952	18	0.00623	4	-2	-3	50	50	100
3	260	50	636.46	3.8260	0.0089	10	-1	-1	15	15	500
4	440	160	669.12	7.9215	1.7E-06	10	-1	-1	25	25	603
5	250	130	590.72	2.4435	0.00906	10	-1	-1	12	12	500

P_{max}: Maximum Capacity (MW)

P_{min}: Minimum Capacity (MW)

a(i), b(i), c(i): Fuel Consumption Coefficient (MBtu, MBtu/MW, MBtu/MW^2)

Min on: Minimum on Time (Hour)

Min off: Minimum off Time (Hour)

InIT: Initial Operational Time (Hour, Minus Means off)

Ramp Up: Ramp-up Rate (MW/min)

Ramp Dn: Ramp-down Rate (MW/min)

St Mbtu: Start-up Mbtu

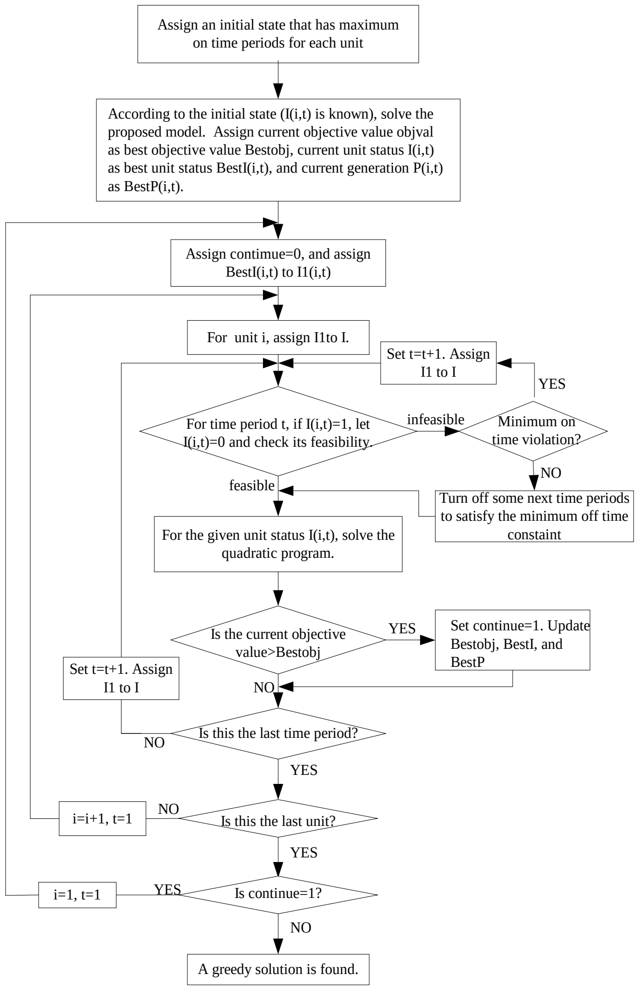

Fig. 7 Flow diagram of the new algorithm for the price-based unit Commitment problem

A. Initialization

In applying the greedy algorithm to the price-based unit commitment, the initial set Q^0 is assigned as all units are on unless it is infeasible. If it is infeasible, it must violate the minimum off time during some beginning time periods. The unit can obtain an initial maximum on time periods by turning off these infeasible time periods. From Table III, it can be seen that initial states of all units have been off for a time interval larger than minimum off time, therefore, all units are on for every time period is a feasible solution. The initial state of I (i, t) is shown in Table IV.

TABLE Ⅳ THE INITIAL STATE OF I(I,T)

	t=0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
i=1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

B. Calculate the Initial Solution

Once an initial feasible 'I (i, t)' is given, this model (eq. (8) ~ (18)) became a quadratic program. Using quadratic programming technique, it can determine the objective value 'objval', and the bidding generation quantity 'P (i, t)' for each unit at every period. Regard the current objective value 'objval', current status of units 'I(i,t)', and generation of all units 'P(i,t)' as 'Bestobj', 'BestI(i,t)' and 'BestP(i,t)' respectively.

C. Iterative Process

The iterative process is to let a time period of one unit off one time if it is on at this time period. If this change violates the minimum on time constraint, this is an infeasible state. But if it violates the minimum off time constraint, it can let it became a feasible state by turning off some more next time periods. For these given states, determine the objective value 'objval' and 'P (i, t)'. If the current objective value 'objval' is greater than 'Bestobj', replace the 'Bestobj' and 'BestP(i, t)' with current 'objval' and 'P(i,t)' respectively, and set index 'continue' equals to 1. The index 'continue' equaling to 1 means that the objective value is still can be improved further, and the iterative process should continue. Units' statuses 'I (i, t)' of the next iteration is based on the 'BestIt (i, t)' of the current iteration. Repeating this process, until no unit turning off at any time period can improve the objective value.

1) Quadratic program [19]:

For a given state of units' status 'I (i, t)', the proposed model, becomes a quadratic program. The general quadratic program can be expressed as (19).

$$\begin{aligned} &\min \quad \frac{1}{2}\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x}+\mathbf{x}^{\mathrm{T}}\mathbf{c} \\ &\text{S.T.} \quad \mathbf{a}_{\mathrm{i}}^{\mathrm{T}}\mathbf{x}=\mathrm{b}_{\mathrm{i}} \quad \mathrm{i}\in \mathrm{E} \\ &\qquad\quad \mathbf{a}_{\mathrm{i}}^{\mathrm{T}}\mathbf{x}\le \mathrm{b}_{\mathrm{i}} \quad \mathrm{i}\in \mathrm{I} \end{aligned} \tag{19}$$

E and I are index sets for equality and inequality constraints. The matrix Q is symmetric and positive semi-definite (if not actually positive definite).

2) Equality Constraints:

A quadratic program is greatly simplified, and can be solved in closed form, if it contains equality constraints only. Thus we consider it in some detail the quadratic program.

$$\begin{aligned} &\min \quad \frac{1}{2}\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x}+\mathbf{x}^{\mathrm{T}}\mathbf{c} \\ &\text{S.T.} \quad \mathbf{A}\mathbf{x}=\mathbf{b} \end{aligned} \tag{20}$$

The Lagrange necessary conditions for this problem are

$$\begin{aligned} &\mathbf{Q}\mathbf{x}+\mathbf{A}^{\mathrm{T}}\boldsymbol{\lambda}+\mathbf{c}=0 \\ &\mathbf{A}\mathbf{x}-\mathbf{b}=0 \end{aligned} \tag{21}$$

If the matrix **Q** is actually positive definite, then an explicit formula for the solution of the system can be easily derived as follows: From the first equation in (21) we have

$$\mathrm{x}=-\mathbf{Q}^{-1}\mathbf{A}^{\mathrm{T}}\boldsymbol{\lambda}-\mathbf{Q}^{-1}\mathbf{c}.$$

Substitution of this into the second equation then yields

$$-\mathbf{A}\mathbf{Q}^{-1}\mathbf{A}^{\mathrm{T}}\boldsymbol{\lambda}-\mathbf{A}\mathbf{Q}^{-1}\mathbf{c}-\mathbf{b}=\mathbf{0},$$

from which we immediately obtain

$$\boldsymbol{\lambda}=-(\mathbf{A}\mathbf{Q}^{-1}\mathbf{A}^{\mathrm{T}})^{-1}\left[\mathbf{A}\mathbf{Q}^{-1}\mathbf{c}+\mathbf{b}\right] \tag{22}$$

and

$$\begin{aligned} \mathrm{x}&=\mathbf{Q}^{-1}\mathbf{A}^{\mathrm{T}}(\mathbf{A}\mathbf{Q}^{-1}\mathbf{A}^{\mathrm{T}})^{-1}\left[\mathbf{A}\mathbf{Q}^{-1}\mathbf{c}+\mathbf{b}\right]-\mathbf{Q}^{-1}\mathbf{c} \\ &=-\mathbf{Q}^{-1}\left[\mathbf{I}-\mathbf{A}^{\mathrm{T}}(\mathbf{A}\mathbf{Q}^{-1}\mathbf{A}^{\mathrm{T}})^{-1}\mathbf{A}\mathbf{Q}^{-1}\right]\mathbf{c}+\mathbf{Q}^{-1}\mathbf{A}^{\mathrm{T}}(\mathbf{A}\mathbf{Q}^{-1}\mathbf{A}^{\mathrm{T}})^{-1}\mathbf{b}. \end{aligned} \tag{23}$$

3) Active Set Method

The general quadratic program with inequality constraints (19) is always solved by an active set method. There is an especially simple version for the case where Q is positive definite. In this version, at iteration k a point $\mathbf{x}_k$ is given that is feasible for all constraints and satisfies all the equality constraints of the current working set W_k. The working set always includes the equality constraints E and possibly some of the inequality constraints I. The quadratic program corresponding to the working set is then defined, by translating to the point $\mathbf{x}_k$, in the form

$$\begin{aligned} &\min \quad \frac{1}{2}\mathbf{d}_k^T\mathbf{Q}\mathbf{d}_k+\mathbf{g}_k^T\mathbf{d}_k \\ &\text{S.T.} \quad \mathbf{a}_i^T\mathbf{d}_k=0, \qquad i\in W_k \end{aligned} \tag{24}$$

where $\mathbf{g}_k=\mathbf{c}+\mathbf{Q}\mathbf{x}_k$.

This program has only equality constraints and can be solved for $\mathbf{d}_k$ by the use of formula (23) above or by other numerically efficient methods. If $\mathbf{d}_k$=**0**, the current point $\mathbf{x}_k$ is optimal with respect to the working set. If $\mathbf{d}_k\neq\mathbf{0}$, and $\mathbf{x}_k+\mathbf{d}_k$ is feasible for all constraints, then $\mathbf{x}_k+\mathbf{d}_k$ becomes the new $\mathbf{x}_{k+1}$. If $\mathbf{x}_k+\mathbf{d}_k$ is not feasible, a search of the form $\mathbf{x}_{k+1}=\mathbf{x}_k+\alpha_k\mathbf{d}_k$ is made, and α_k is selected as large as possible to maintain feasibility. The general move is therefore $\mathbf{x}_{k+1}=\mathbf{x}_k+\alpha_k\mathbf{d}_k$, where

$$\alpha_k=\underset{\mathbf{a}_i^T\mathbf{d}_k>0}{\text{minimum}}\left\{1,\ \frac{b_i-\mathbf{a}_i^T\mathbf{x}_k}{\mathbf{a}_i^T\mathbf{d}_k}\right\} \tag{25}$$

V. TEST RESULTS

In this paper, the 5-unit system and 36-unit system are used to test the proposed model. Units' parameters of the 5-unit system are shown in Table III. Parameters of the 36-unit system are available in [20]. The fuel price is 3$/MBtu. In order to consider uncertainties of energy prices, possible energy price scenarios as shown in Fig. 6 have been taken into account. Some test results are demonstrated below.

In Table V, Unit 2 is shut down in the 4th and 5th time period in the first iteration for the 5-unit system. The second iteration determines Unit 2 should still be off at the 6th time period. Executing the iteration process shown in Fig. 7, a converging solution will be obtained when no unit turning off could improve the objective function. Tables VI and VII show the final units' statuses and power generations respectively. Fig. 8 shows the improving process of GENCO's profit. The final profit that GENCO could obtain is 503,705 dollars.

TABLE Ⅴ PARTS OF ITERATING PROCESS (5-UNIT SYSTEM)

Iteration	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**·····**
Unit	-	2	2	2	2	2	2	2	2	2	2	2	·····
Off period	-	4,5	6	7	22,23	8	9	20,21	19	10	18	11	·····

TABLE Ⅵ OPTIMAL UNITS' STATUSES (5-UNIT SYSTEM)

	t=0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**
i=1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

TABLE Ⅶ OPTIMAL UNITS' POWER GENERATIONS (5-UNIT SYSTEM)

	t=0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Unit 1	off	off	off	off	off	off	off	Off	off	off	off	off
	t=12	13	14	15	16	17	18	19	20	21	22	23
	off	off	off	off	off	off	off	Off	off	off	off	off
	t=0	1	2	3	4	5	6	7	8	9	10	11
Unit 2	25	25	25	25	off	off	off	Off	off	off	off	off
	t=12	13	14	15	16	17	18	19	20	21	22	23
	off	off	100	96.5	100	100	off	Off	off	off	off	off
	t=0	1	2	3	4	5	6	7	8	9	10	11
Unit 3	260	260	260	260	260	260	260	260	260	260	260	260
	t=12	13	14	15	16	17	18	19	20	21	22	23
	260	260	260	260	260	260	260	260	260	260	260	260
	t=0	1	2	3	4	5	6	7	8	9	10	11
Unit 4	440	440	440	440	440	440	440	440	440	440	440	440
	t=12	13	14	15	16	17	18	19	20	21	22	23
	440	440	440	440	440	440	440	440	440	440	440	440
	t=0	1	2	3	4	5	6	7	8	9	10	11
Unit 5	250	250	250	250	250	250	250	250	250	250	250	250
	t=12	13	14	15	16	17	18	19	20	21	22	23
	250	250	250	250	250	250	250	250	250	250	250	250

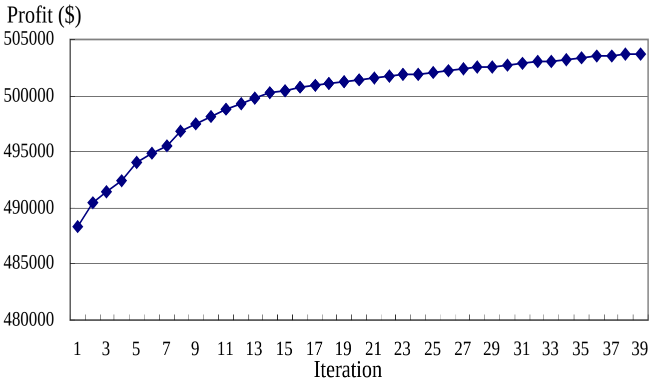

Fig. 8 the improving process of profit (5-unit system)

Table VIII shows parts of the iterating process of the 36-unit system. Due to the minimum off time constraint, Unit 25 and 26 must remain the off state till the 8th time period initially. Unit 24 is shut down between the 6th and 9th time period in the first iteration. The second iteration determines Unit 23 should also be shut down at the same time intervals. Executing the iterating process shown in Fig. 7, a converging solution will be obtained when no unit turning off could improve the objective function. Table IX shows parts of the final results. Fig. 9 shows the improving process of the GENCO's profit. The final profit that GENCO could obtain is 1.44874×10^6 dollars.

TABLE Ⅷ PARTS OF ITERATING PROCESS (36-UNIT SYSTEM)

Iteration	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**…..**
Unit	25,26	24	23	27	22	26	25	24	23	…..
Off period	0-7	6-9	6-9	6-9	6-9	20-23	20-23	20-23	20-23	……

TABLE Ⅸ OPTIMAL UNITS' POWER GENERATIONS (36-UNIT SYSTEM)

Unit 1-7	t=0	1	2	3	4	5	6	7	8	9	10	11
	off	off	off	off	off	off	off	off	off	off	off	off
	t=12	13	14	15	16	17	18	19	20	21	22	23
	off	off	off	off	off	off	off	off	off	off	off	off
Unit 8-11	t=0	1	2	3	4	5	6	7	8	9	10	11
	off	off	off	off	off	off	off	off	off	76	76	76
	t=12	13	14	15	16	17	18	19	20	21	22	23
	76	76	76	76	76	76	76	76	76	76	76	off
Unit 12	t=0	1	2	3	4	5	6	7	8	9	10	11
	25	25	25	25	25	off	off	off	off	off	off	off
	t=12	13	14	15	16	17	18	19	20	21	22	23
	off	82	96	100	96	off	off	off	off	off	off	off
Unit 13	t=0	1	2	3	4	5	6	7	8	9	10	11
	25	25	25	25	25	off	off	off	off	off	off	off
	t=12	13	14	15	16	17	18	19	20	21	22	23
	off	off	off	58	50	57	59	off	off	off	off	off
Unit **14**	t=0	1	2	3	4	5	6	7	8	9	10	11
	25	25	25	25	25	off	off	off	off	off	off	off
	t=12	13	14	15	16	17	18	19	20	21	22	23
	off	75	90	97	90	off	off	off	off	off	off	off

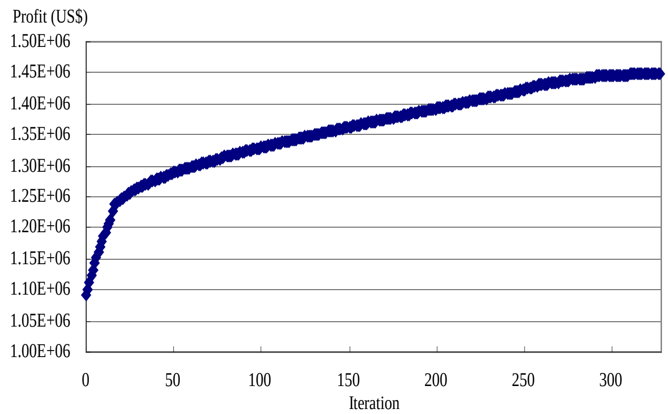

Fig. 9 The improving process of profit (36-unit system)

In order to verify the proposed model to obtain more profits than those that the crisp model can do, the data of load demand and energy prices between 2005/06/01 and 2005/06/07 are reserved. Using these real energy prices, the real profit can be compared between the crisp model and the proposed fuzzy model. Tables Ⅹ and XI compare the real profits obtained by the regression model and the proposed fuzzy model for the 5-unit system and 36-unit system respectively. From these tables, it can be seen that the proposed model considering uncertainties can obtain more profits.

TABLE Ⅹ COMPARISON OF THE REAL PROFIT BETWEEN THE REGRESSION MODEL AND THE PROPOSED FUZZY MODEL (5-UNIT SYSTEM)

Date	Regression model	The proposed model
2005/06/01	4.9760×10^5	4.9805×10^5
2005/06/02	4.5707×10^5	4.5808×10^5
2005/06/03	4.4780×10^5	4.4623×10^5
2005/06/04	5.6284×10^5	5.6284×10^5
2005/06/05	6.1347×10^5	6.1453×10^5
2005/06/06	8.3116×10^5	8.3195×10^5
2005/06/07	9.2643×10^5	9.3117×10^5

TABLE Ⅺ COMPARISON OF THE REAL PROFIT BETWEEN THE REGRESSION MODEL AND THE PROPOSED FUZZY MODEL (36-UNIT SYSTEM)

Date	Regression model	The proposed model
2005/06/01	1.4580×10^6	1.4729×10^6
2005/06/02	1.2191×10^6	1.2352×10^6
2005/06/03	1.1858×10^6	1.1878×10^6
2005/06/04	1.6828×10^6	1.6828×10^6
2005/06/05	1.9120×10^6	1.9408×10^6
2005/06/06	2.9071×10^6	2.9223×10^6
2005/06/07	3.3465×10^6	3.3733×10^6

VI. CONCLUSIONS

In the deregulated power market, GENCOs regard obtaining maximum benefit as their object instead of satisfying the customer's demand. In this new environment, GENCOs should face uncertainties exiting in power market to pursue their profits. This paper proposes a new model considering uncertainties of energy prices. An efficient algorithm that combines the greedy algorithm and quadratic program is introduced to solve this model. Test results show this proposed model can obtain more profits than those that the crisp model can do.

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